SRR & CVR GOVT. DEGREE COLLEGE

(Autonomous) NAAC 'B^{+}' Grade

DEPARTMENT OF MATHEMATICS

II B.Sc. MATHEMATICS

PAPER – III, SEMESTER – III

ABSTRACT ALGEBRA

MODEL QUESTION PAPER

Time: 3 Hours **Max.Marks:60**

Section-A

Answer any FIVE Questions **5x4 = 20 M**

1. Show that, in a group, inverse of any element is unique.

2. Prove that the set of integers Z is an abelian group for the operation * defined by a*b = a+b+1, $\forall$ a, b $\in z$.

3. Show that intersection of two subgroups is also a subgroup.

4. If H is any subgroup of a group G, then prove that $H^{-1} = H$.

5. Show that every subgroup of an abelian group is normal.

6. Prove that the homomorphic image of a group is a group.

7. Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 1 & 2 \end{pmatrix}$ as the product of disjoint cycles.

8. If G is a cyclic group of order n and 'a' is a generator of G. Show that a^m is a generator of G if and only if (m, n)=1.

9. Prove that every filed is an Integral domain.

10. Prove that the characteristic of a Boolean ring is 2.

Section-B

Answer any ALL Questions. **5x8 = 40 M**

11. (a). If G is a group and a, b ∈ G, then prove that the equations ax = b and ya = b have unique solutions in G.

(or)

(b). Prove that the set G or real numbers other than '-1' with operation * such that a*b = a+b+ab, ∀ a, b ∈ G is an abelian group.

12. (a). State and prove Lagrange's theorem for finite groups.

(or)

(b). Let H be a subgroup of a group G and a, b ∈ G, then prove that

(i) Ha = Hb ⇔ ab^{-1}∈H, (ii) aH = bH ⇔ $a^{-1}b \in H$.

13. (a) If H is a subgroup of G and N is normal subgroup of G, then show that

(i) H∩N is a normal subgroup of H and (ii) N is a normal subgroup of HN.

(or)

(b). Prove that every homomorphic image of a group G is isomorphic to some quotient group of G.

14. (a). Let 'S_n' be a symmetric group of n symbols and left 'A_n' be the group of even permutations, then show that 'A_n' is a normal is 'S_n' and $O(A_n) = \frac{1}{2}n!$.

(or)

(b). Prove that every subgroup of a cyclic group is cyclic.

15. (a). Prove that the characteristic of an Integral domain is either zero or prime number.

(or)

(b). Prove that Union of two subrings is also a subring if and only if one is contained another.